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AERODYNAMIC COMPUTATION OF GLIDERS.

By M. Schrenk.

From "Flugsport," May 24, 1922.

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The remarkable soaring-flight performances of last autumn and the large prizes now offered lead us to expect considerable improvement in the maximum performances this year. Every interested constructor must therefore strive all the more to obtain insight by computation into the most favorable proportions for his gliders.

There is indeed no perfect glider theory, that has been confirmed by scientific experiments. One follows the theory of Knoller-Betz** and is successful, another adopts adjustable wings (Harth) with like success, and a third makes record performances with an ordinary airplane, without special theory. We may however lay down one principle, which is of equal significance for every kind of construction, and which will serve as the starting point for all computations: The energy requirement of an aircraft for gliding flight must be the minimum.

In the following discussion, a knowledge of the theoretical principles of airplane construction is assumed, as presented in detail by Vogt and Lippisch in Nos. 7 and 10-19 of the 1919 volume of this publication. A few quantities will however be otherwise designated, in accordance with the Göttingen symbols.

* From "Flugsport," May 24, 1922, pp. 170-177.

** Dr. A. Betz, of the Göttingen Aerodynamic Institute, had already computed, in 1912, that an airfoil capable of turning about its transverse axis may reap an advantage from a periodic change in the direction of the wind (Zeitschrift für Flugtechnik und Motorluftschiffahrt, 1912, pp. 269-272).

The above-mentioned requirement of minimum energy is self-evident. Any airplane must have a certain speed for each angle of attack, in order to maintain horizontal flight. This produces a certain head resistance, which is the sum of the resistance of the wings and the "parasite" resistance of the non-lifting parts. This resistance D, at the speed per second v, must be overcome by a flight Power energy Dv in kg.m/sec. If, with an engineless aircraft, we designate horizontal flight as soaring flight, and consider, further, that the energy of such an aircraft is obtained in some way from the energy of moving air, we realize that soaring flight will be just so much more easily attained as the energy to be obtained from the wind is smaller. It is fundamentally the same, whether this energy is acquired through the skill of the pilot or through the better adaptation of the aircraft itself.

Instead of expressing this energy requirement by our usual symbols C_L, C_D, S, and W, we will represent these relations in another manner (Fig. 1). We will let v_y represent the vertical descending speed per second of the aircraft in gliding flight. If the aircraft were now raised the distance v_y per second by the weight W, it would continue its flight without loss of altitude. This process can be imagined as divided at will and leads quite simply to the representation of the flight energy as the product of W times v_y. Since, however, the weight W of the aircraft remains constant, any lessening of the required flight energy amounts to the same thing as a lessening of the descending speed. In other words: "The descending speed must be reduced as much as possible."

In gliding flight, the air speed of the aircraft is computed (See also "Flugsport," 1919, p. 522) from

$$L = W \cos \varphi = C_L S \frac{\rho}{2} v^2$$

in which $\rho = \frac{\gamma}{g}$ the air density. At small gliding angles (5 to 10°) of our best airplanes, $\cos \varphi = 1$ is accurate enough and hence

$$L = W = C_L S \frac{\rho}{2} v^2$$

from which we obtain the wind speed

$$v = \sqrt{\frac{\overset{W}{g}}{S} \frac{2}{\rho} \frac{1}{C_L}}$$

The descending speed is $v_y = v \sin \varphi$

Again, with sufficient accuracy,

$$\sin \varphi = \tan \varphi = \frac{D}{L} = \frac{C_D}{C_L}$$

Hence

$K = f(A)$

$$v_y = \frac{C_D}{C_L} \sqrt{\frac{W}{S} \frac{2}{\rho} \frac{1}{\underset{C_L}{C_D}}} = \sqrt{\frac{W}{S} \frac{2}{\rho} \frac{1}{C_L^3 / C_D^2}} = \left(K \frac{1}{C_L A}\right)^{\frac{1}{2}}$$

To leave this quantity as small as possible is the object of the computation which the builder must make before undertaking the construction of a glider.

There are two factors which determine the value of v_y: the wing load W/S and the climbing coefficient C_L^3/C_D^2 of the aircraft. Any diminution of the wing load produces the same effect as an increase in the climbing coefficient on the descending speed.

Both factors are intimately dependent on each other. These factors will first be considered separately. The rest of the article will then show mathematically all the mutual relations and influences.

The wing load W/S depends on the size of the wings and the ability of the builder to save as much weight as possible. Its value lies ordinarily between 6 and 12 kg/m². To go much higher is forbidden by the increased difficulty in starting. Any further diminution is improbable, on account of the general weight relations and would necessitate disproportionately large wings.

The climbing coefficient C_L^3/C_D^2 demands our principal attention, since the determination of a wing section,* with regard to its aerodynamic qualifications, conforms to the behavior of the function C_L^3/C_D^2, which has to do with the coefficients of the wing section under consideration. The values of this function vary directly as the lift coefficients of the wing section and inversely as the head resistance of the whole aircraft for these high values of C_L.**

In connection with wing resistance, the aspect ratio $\frac{1}{\lambda} = \frac{S}{b^2}$. (In rectangular wings this equals c/b = chord/span). The separation of the supporting vortices near the wing tips produces a marginal or induced drag, which depends on the aspect ratio and the square of the lift, according to the parabola equation:

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* The most recent and reliable wing-section researches are contained in the "Berichten der aerodynamischen Versuchsanstalt Göttingen" for 1921. This book is indispensable for the scientific constructor.

** Of course, in all these computations, C_L and C_D stand for the whole aircraft. In this case Vogt and Lippisch write Ca_g and Cw_g, which were not adopted here, in order to avoid the confusion of so many appendages.

Induced drag $= C_{D_i} = \frac{C_L^2}{\pi} \frac{S}{b^2}$

The Göttingen data were all obtained from models having an aspect ratio of 5. For converting these results to any desired aspect ratio λ, advantage is taken of the circumstance that, in all existing aspect ratios, the wing resistance, due to the viscosity and internal friction of the air, remains the same. If its coefficient is C_{D_o} (Vogt: C_{D_f}), (the index representing the aspect ratio):

$$C_{D_o} = C_{D_5} - \frac{C_L^2}{\pi} \frac{1}{5} = C_{D_\lambda} - \frac{C_L^2}{\pi} \lambda \qquad \lambda = \frac{1}{AR} = \frac{b^2}{S}$$

from which is obtained the conversion formula

$$C_{D_\lambda} = C_{D_5} - \frac{C_L^2}{\pi} \left(\frac{1}{5} - \lambda\right)$$

A few brief computations will convince the reader of the importance of the induced drag.*

Since the climbing coefficient stands for the whole airplane, it includes, in addition to the wing resistance, the parasite resistance of all non-supporting parts with the areas f_1, f_2 .... and the coefficients C_{Dp_1}, C_{Dp_2} .... Taken as a whole and referred to the wings (as is customary in airplane computations), it is given the coefficient

$$C_{Dp} = \frac{\Sigma f_n C_{Dp_u}}{S}$$

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* It should be noted that the angle of attack also changes for every aspect ratio. To every angle of attack α_∞, which a wing with an aspect ratio of $1 : \infty$ requires for producing a certain lift, there is added a so-called induced angle of attack. There is then obtained, as above, from

$$\alpha_\infty = \alpha_5 - \frac{C_L}{\pi} \frac{1}{5} = \alpha_\lambda - \frac{C_L}{\pi} \lambda$$

the conversion formula for the angle of attack:

$$\alpha_\lambda = \alpha_5 - \frac{C_L}{\pi} \left(\frac{1}{5} - \lambda\right)$$

This value is introduced into the computation as a constant for all the angles of attack, since its variations (not inconsiderable under some conditions) cannot be satisfactorily determined. Evidently this value is the smallest when the aircraft is moving through the air in the direction of its principal axis. From this fact, there follows the structural condition, that the wings be given the angle of incidence to the principal axis which yields the maximum value for C_L^3/C_D^2, in order that the parasite resistances will be the smallest at the slowest descending speed.

In order to make this clearer, the functions C_L^3/C_D^2 are plotted against the angle of attack for a monoplane with the Göttingen wing sections 376 and 441* with aspect ratios of 5 and 10 (Fig. 2). In the numerical computation, a C_{Dp} of 1.5 was adopted for the airplane with the self-supporting thick wing 441 and a C_{Dp} of 2.0 for the airplane with the thin wing section 376, on account of the parasite resistance of the truss wires.

The curves of the climbing coefficients show characteristic differences. Increasing the aspect ratios more than doubles the values of C_L^3/C_D^2 for both wing sections, in which connection the greater lift of wing section 441 first becomes noticeable for the larger aspect ratio. Soaring flight is of great importance for practical utilization and, furthermore, the extension of the range of the angles of attack for use in the descending speed is small. In this respect the thick wing section is much better than the thin

* Attention is called to the fact that in the representation of the air forces in the Lilienthal polar diagram, the coefficients are multiplied by 100, so that

$$C_a = 100\ c_a \text{ and } C_w = 100\ c_w.$$

one, especially with a large aspect ratio. It is likewise better with respect to the maximum of the curve, which, for a thin wing section, lies much nearer the limit of the utilizable angles of attack (about 10°), than for the thick wing section (about 6°). The latter gives a much greater degree of safety in the event of stalling, which may happen at any instant in soaring flight, as a result of changes in the direction of the wind. On the other hand, with the improvement of the aspect ratio, both wing sections become more sensitive to changes in the angle of attack, i.e., it becomes more difficult for the pilot to maintain the most favorable position, of the aircraft.

Any airplane builder may add to these examples at his discretion. When he has gone over the process of computation a few times, he will acquire a certain intuition as to which wing section is better for his aircraft.

The coefficient of glide $C_L/C_D = \cot \varphi$ affects the soaring-flight computation only when, in addition to soaring ability, a horizontal flight path is desired, in order to be able to start from gentle slopes or glide long distances. Increasing $C_L{}^3/C_D{}^2$ always improves the gliding coefficient, wherefore a glider, for which a horizontal flight path is desired, must be developed more in this direction than in the direction of a small wing load.

In order to obtain an aerodynamically good glider, there would be, according to the foregoing explanations, nothing else to do, but to choose a very small wing load and provide the resulting surface area with a strong, lift-generating wing section and large

aspect ratio. Still other considerations, as in every technical problem, reduce the solution to a compromise between opposing demands. First of all comes the limitation of the span, for reasons of structural safety, especially in the now popular monoplanes with self-supporting wings, then maneuverability, and carrying capacity. How far these can be improved has not yet been determined, therefore the constructor still has a wide field.

Hitherto, it was impossible to understand the relations between the two factors W/S and $C_L{}^3/C_D{}^2$, whose unknown mutual influence reduced the whole computation to tentative tests. In order to obtain a clear conception of the problem, the quantities are largely separated into their elements. S and C_L, however, cannot be divided. C_D and W become respectively

$$C_D = C_{D_o} + C_{Dp} + C_{D_i} = C_{D_o} + \frac{fC_{Dp}}{S} + (\kappa)\ \frac{C_L{}^2}{\pi}\ \frac{S}{b^2}{}^{*} \quad \text{and } W=K+wS.$$

Here, for the sake of simplicity, fC_{Dp} denotes the surface of parasite resistance, which was indicated above by $\Sigma f_n\ C_{Dp_n}$. W was divided into the weight K of all non-lifting parts (including the pilot) and into the weight wS of the wings, in which w denotes the unit weight of wing area. The descending speed is then

$$v_y = \frac{4}{C_L}\left(C_{D_o} + \frac{fC_{Dp}}{S} + (\kappa)\ \frac{C_L{}^2}{\pi}\ \frac{S}{b^2}\right)\sqrt{\frac{K}{C_L S} + \frac{w}{C_L}}$$

(whereby $2/\rho = 16$ at sea level).

* (κ) comes into biplane computations. It gives the relation of the induced resistances of a monoplane to those of a biplane of like S/b^2 and depends on the ratio of the spans of the upper and lower wings and the distance separating them. More accurate details are given in the "Technische Berichte III der Flugzeugmeisterei," p. 309: "Der induzierte Widerstand von Mehrdeckern" (Induced Drag of Multiplanes) by L. Prandtl.

In the above expression, all the quantities except S, are known individually or can be determined. C_L, on the basis of the polars of the chosen wing section and the probable aspect ratio, is so chosen that it lies in the vicinity of the probable maximum of C_L^3/C_D^2. Thereby C_{D_o} remains constant. fC_{D_p} can be computed in known manner from the drawing and is only slightly dependent on the wing area to be employed. The same holds good for K. Finally, the unit weight of surface w, according to construction and aspect ratio, varies between 1.5 and 2 kg/m². This value, at the discretion of the constructor, can also be introduced into the computation and be regarded as constant. It has only a slight effect on the result. In order to obtain the surface area which will reduce the descending speed to a minimum, a simple minimal computation must be carried through.

$$\frac{d\,v_y}{d\,S} = \varphi\,(S) = 0$$

gives, after a few transformations,

$$(\kappa)\ 2w\ C_L^2\ S^3 + (\kappa)\ K\ C_L^2\ S^2 - \pi\,b^2\ (2\ w\ f\ C_{D_p} + KC_{D_o})S - \pi\,b^2 Kf\ C_{D_p} = 0$$

This equation of the third degree in S gives the desired area and clearly shows the influences of the different elements. The wing surface increases with increasing K, f C_{D_p}, and C_{D_o}, and decreases with increasing C_L and w.

The wing surface also increases with increasing span. In order to discover how the chord c is affected, the equation is transformed, with the aid of S = b c, and arranged according t

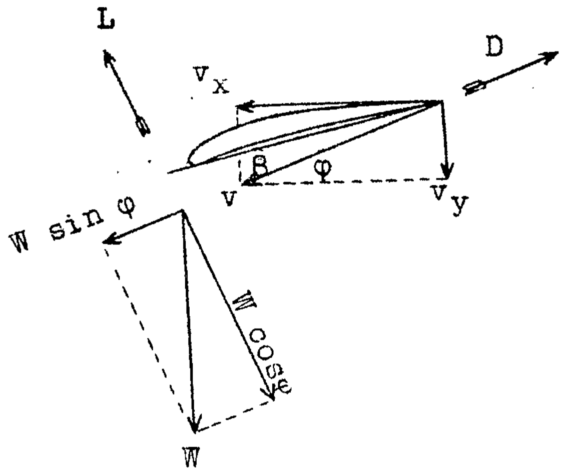

Fig. 1.

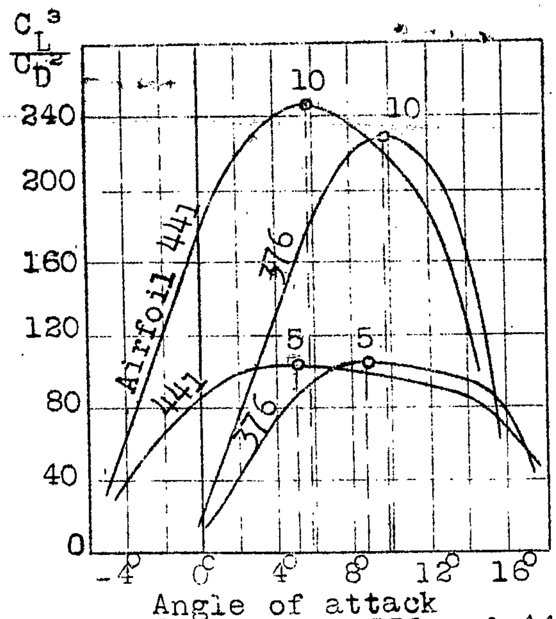

Fig. 2-Airfoils 376 and 441.
Aspect ratios 5 and 10.

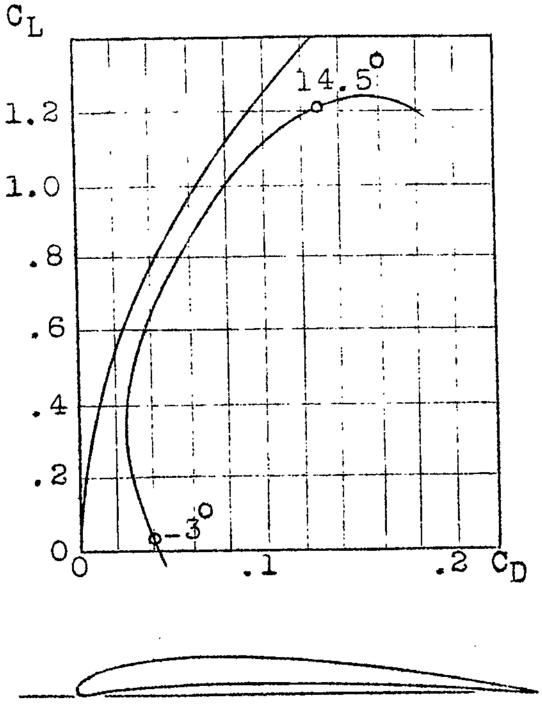

Fig. 3-Airfoil 376.
Aspect ratio 5.

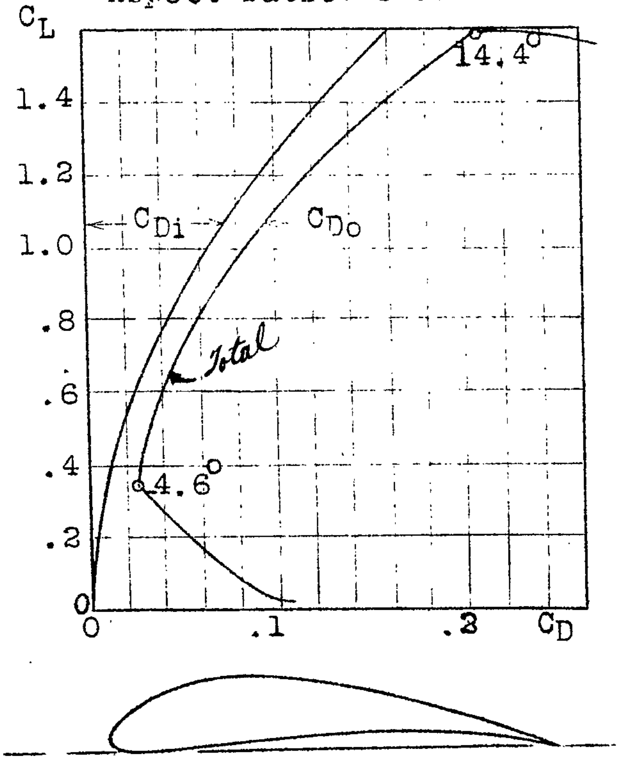

Fig. 4- Airfoil 441.
Aspect ratio 5.

Figs. 1-2-3-4. encl